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Thermal-stress analysis for a strip of finite width containing a stack of edge cracks

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Abstract The thermal-stress problem of an infinite strip containing an infinite row of periodically distributed edge cracks normal to its edge is investigated. The surrounding temperature adjacent to the crack-containing edge is assumed to be cooled suddenly to simulate the thermo-shock condition. By the superposition principle, the formulation leads to a mixed-boundary-value problem, with the negating tractions derived from the thermal stresses of a crack-free infinite strip. An integral equation is obtained and solved numerically. The effect on the SIFs (stress-intensity factors) due to the presence of periodically distributed cracks in an infinite strip is delineated. The normalized SIFs increase as the stacking cracks separate, due to the reduction of the shielding effect. After a characteristic time period, the negating tractions along the crack faces become almost linear. The SIF solutions under the considered crack geometry are worked out in detail for the case of linear traction loading.

Keywords Cracks · Integral transform · Stress-intensity factors · Thermal loads

1 Introduction

The thermo-mechanical analysis under thermal-shock condition is of vital importance for the primary system of reactor structures, especially in the presence of pre-existing cracks. The cracking of embrittled solids, due to thermal transients, has long been recognized as an intrinsic failure mechanism. The transient thermal analysis for an elastic strip with cracks has attracted many investigators [1–4]. The problems of an edge crack in an infinite strip were investigated under different thermal boundary conditions. Shindo and Atsumi studied the case of an infinite plate containing an infinite row of parallel cracks under thermal stresses [5,6]. Davidson examined the problem of a collinear array of Griffith cracks contained in an infinite strip [7].

Under the thermal-shock condition, the cooling phase in the plate surface is generally regarded as the most severe [3,4], while in the surface-heating phase the edge cracks along the surface may partially close. Qing and

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Yang [8] analyzed the problem of a semi-infinite plate containing an infinite row of periodically distributed cracks normal to its edge. The present paper deals with an infinite strip of finite width with an infinite row of periodically distributed edge cracks. The issue concerns the determination of SIFs when the strip is subjected to different thermal boundary conditions. The strip occupies the region of $0 \leq X \leq a$ and $-\infty < Y < +\infty$, and deforms in a plane-strain or plane-stress condition. The cracks, being thermally insulated and traction-free, are configured as a periodic array defined by $0 \leq X \leq c_1$, $Y = 2Nb_1$, with $N = 0, \pm 1, \pm 2, \ldots$ (as shown in Fig. 1). The plate is assumed to be composed of homogeneous, linear and isotropic material and the inertia effect is neglected. The study on dynamic thermo-elasticity in [9] validated this assumption. The thermo-elastic coupling, as well as the temperature dependence of thermo-elastic constants, are ignored for simplicity.

The solution of the problem is obtained by using the superposition technique, developed from the following thought experiments. First, the thermal stresses within the uncracked strip are evaluated. Second, the perturbation problem for the cracked strip is formulated with the crack-face tractions negating the thermo-elastic stresses determined in part one. The problem is reduced to an integral equation through the integral-transform technique, which is then solved numerically. Results are obtained for various loadings and geometry parameters.

2 Heat conduction and thermal stresses

Consider a homogeneous, isotropic and thermo-elastic plate in the form of an infinite strip containing an infinite stack of periodically distributed edge cracks, as shown in Fig. 1. The crack surfaces are thermally insulated. The strip and the surrounding environment are initially at uniform temperature T_0. Due to thermal loading, the surrounding temperature adjacent to the crack-containing edge is suddenly cooled from T_0 to T_∞. The temperature differences from the edge are defined as

$$\Theta(X, \tau) = T(X, \tau) - T_0, \quad \Theta_1 = T_\infty - T_0, \quad \Theta_2 = 0,$$

where Θ_1 and Θ_2 denote the surrounding temperature differences adjacent to the boundaries $X = 0$ and $X = a$, respectively. To normalize the problem, the following quantities are introduced:

$$x = X/a, \quad t = D\tau/a^2, \quad \theta(x, t) = \Theta(x, t)/\Theta_1,$$

where D is the thermal diffusivity. Then the governing heat-conduction equation is given by

$$\frac{\partial^2\theta(x, t)}{\partial x^2} = \frac{\partial\theta(x, t)}{\partial t}. \tag{1}$$

The initial and boundary conditions are specified as

$$\theta(x, 0) = 0, \quad \begin{cases} -\theta'(0, t) + ah_1\theta(0, t) = ah_1 \\ \theta'(1, t) + ah_2\theta(1, t) = 0 \end{cases}, \tag{2}$$

where h_1 and h_2 are the thermal exchanging coefficients between the strip and the surrounding environments at $X = 0$ and $X = a$, respectively.

After lengthy but straightforward algebra, the temperature field of this heat-conduction problem can be obtained in the following form:

$$\theta(x, t) = \frac{h_1}{h_1 + h_2 + ah_1h_2}(1 + ah_2 - ah_2x) - \sum_{i=1}^{\infty} \frac{2h_1^2a^3(h_1 + h_2)k_i(\sin(k_ix) + k_i\cos(k_ix)/ah_1)}{(a^2h_1^2 + k_i^2)(ak_i^2(h_1 + h_2) + (a^2h_1h_2 + k_i^2)\sin^2(k_i))}\mathrm{e}^{-k_i^2t} \tag{3}$$

where the eigenvalues k_i are determined by the following equation

$$\tan(k_i) = \frac{ak_i(h_1 + h_2)}{k_i^2 - a^2h_1h_2}, \quad i = 1, 2, 3 \ldots . \tag{4}$$

Notice that if $h_i \to +\infty\,(i=1,2)$, the thermal boundary conditions are reduced to $\theta=\theta_i$; and if $h_i \to 0\,(i=1,2)$, the thermal boundary conditions are reduced to the thermally insulating boundary condition, namely $\partial\theta_i/\partial n=0$. The thermal-stress field can be obtained (in the spirit of [9]) for the uncracked infinite strip as follows

$$\sigma_Y^T(X,\tau)=-\sigma_0^T\theta(x,t). \tag{5}$$

In (5), the thermal stress is scaled by $\sigma_0^T=\alpha\overline{E}\Theta_1$, where $\overline{E}$ is $E/(1-\upsilon)$for the plane stress case, and is $E/(1-2\upsilon)$ for the plane strain case; with E, υ and α denoted for Young's modulus, the Poisson ratio, and the coefficient of thermal expansion, respectively.

3 Crack problem

3.1 Formulation

Consider an infinite strip containing an infinite row of edge cracks normal to its edge (shown in Fig. 1). Denote the horizontal and vertical displacements as U and V. The following normalized quantities are introduced:

$$x=X/a,\quad y=Y/a,\quad u(x,y)=U(X,Y)/a,\quad v(x,y)=V(X,Y)/a.$$

The governing equations for the normalized displacements u and v of the plane elasticity are

$$(\kappa-1)\nabla^2u+2\left(\frac{\partial^2u}{\partial x^2}+\frac{\partial^2v}{\partial x\partial y}\right)=0,\quad(\kappa-1)\nabla^2v+2\left(\frac{\partial^2u}{\partial x\partial y}+\frac{\partial^2v}{\partial y^2}\right)=0, \tag{6}$$

where $\kappa=3-4\nu$ for the plane strain case, and $\kappa=(3-\nu)/(1-\nu)$ for the plane stress case. The Laplace operator is denoted by $\nabla^2=\partial/\partial x^2+\partial/\partial y^2$. The stress-displacement relations for the plane problem are:

$$s_x=\frac{\mu}{\kappa-1}\left((\kappa+1)\frac{\partial u}{\partial x}+(3-\kappa)\frac{\partial v}{\partial y}\right),\quad s_y=\frac{\mu}{\kappa-1}\left((\kappa+1)\frac{\partial v}{\partial y}+(3-\kappa)\frac{\partial u}{\partial x}\right),\quad s_{xy}=\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right), \tag{7}$$

where, μ denotes the shear modulus, and s_x, s_y, s_{xy} denote various components of stress.

A representative cell is selected as in Fig. 2. In accordance with the periodicity and the symmetry of the crack problem, the boundary conditions of the representative cell are

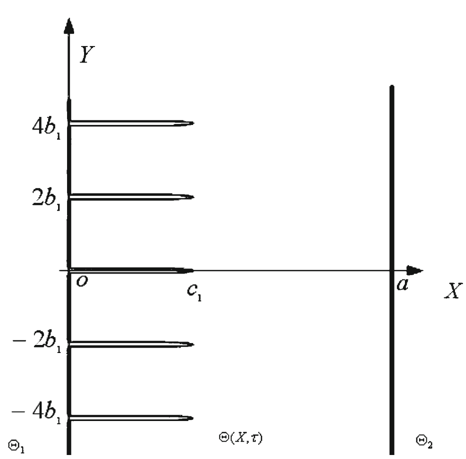

Fig. 1 Configuration of an array of edge cracks and the temperature differences from the edge

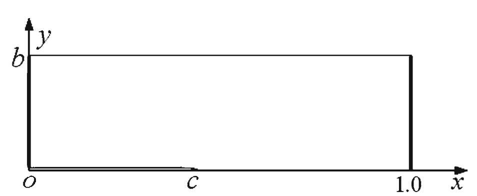

Fig. 2 A representative cell

$$x = 0: \quad s_x = s_{xy} = 0, \tag{8}$$

$$x = 1: \quad s_{xy} = u = 0, \tag{9}$$

$$y = b: \quad s_{xy} = v = 0, \quad s_{xy}(x, 0) = 0, \tag{10}$$

$$y = 0: \quad \begin{cases} v(x, 0) = 0 & c < x < 1, \\ s_y(x, 0) = p(x) & 0 \le x \le c, \end{cases} \tag{11}$$

where $p(x)$ represents the load along the crack faces.

3.2 Solution of the crack problem

The differential equations (6) are solved by expressing u and v in terms of Fourier series of the following forms [8]

$$\begin{cases} u(x, y) = f_0(x) + \sum_{i=1}^{\infty} f_i(x)\cos(\beta_i y) + \sum_{j=1}^{\infty} h_j(y)\sin(\alpha_j x), \\ v(x, y) = q_0(y) + \sum_{i=1}^{\infty} g_i(x)\sin(\beta_i y) + \sum_{j=1}^{\infty} q_j(y)\cos(\alpha_j x), \end{cases} \tag{12}$$

where $\alpha_i = i\pi$ and $\beta_j = j\pi/b$. Substituting (12) in (6), one obtains a system of ordinary differential equations for the unknown functions $f_0, f_i, g_0, q_0, q_j, h_j$ $(i, j = 1, 2, \ldots)$. After lengthy but straightforward computations, the expressions for the displacements and stresses are obtained as follows:

$$\begin{aligned} u(x, y) &= u_0 + u_1 x + \sum_{i=1}^{\infty}((L_i + M_i x)\sinh(x\beta_i) + (N_i + P_i x)\cosh(-x\beta_i))\cos(\beta_i y) \\ &\quad + \sum_{j=1}^{\infty}((Q_j + R_j y)\sinh(\alpha_j y) + (S_j + T_j y)\cosh(\alpha_j y))\sin(\alpha_j x), \\ v(x, y) &= v_0 + v_1 y + \sum_{i=1}^{\infty}([(L_i + M_i x + \kappa P_i/\beta_i))\cosh(x\beta_i) - (W_i + P_i x + \kappa M_i/\beta_i)\sinh(x\beta_i))\sin(\beta_i y) \\ &\quad - \sum_{j=1}^{\infty}(S_j + T_j y - \kappa R_j/\alpha_j)\sinh(\alpha_j y) + (Q_j + R_j y - \kappa T_j/\alpha_j)\cosh(\alpha_j y)]\cos(\alpha_j x), \end{aligned} \tag{13}$$

where $u_0, u_1, L_i, M_i, N_i, P_i$ and $v_0, v_1, Q_j, R_j, S_j, T_j$ are the functions related to i and j, respectively. Routinely, one may derive the stress fields. After lengthy but straightforward algebra, one obtains the following integral equation:

$$\int_0^c \phi(t)F(x, t)\mathrm{d}t = p(x), \tag{14}$$

where

$$\phi(x) = \frac{4\mu}{1+\kappa}\frac{\partial v(x, 0)}{\partial x}, \quad (0 \le x \le c)$$

$$\begin{aligned} F(x, t) &= \frac{2}{b}t + \sum_{i=1}^{\infty} 2\coth(bi\pi)(1 + 2bi\pi \csc h(2bi\pi))\cos(i\pi x)\sin(i\pi t) \\ &\quad - \frac{8b}{\pi}\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\frac{ij^2\sin(i\pi t)}{(b^2 i^2 + j^2)^2(b + 2j\pi \csc h(\frac{2j\pi}{b}))}\left\{\sinh\left(\frac{j\pi x}{b}\right)\left(j\pi x - b\tanh\left(\frac{j\pi}{b}\right)\right)\right. \\ &\quad \left. + \cosh\left(\frac{j\pi x}{b}\right)\left(2j\pi \csc h\left(\frac{2j\pi}{b}\right) + j\pi x\tanh\left(\frac{j\pi}{b}\right) - b\right)\right\} \end{aligned}$$

Now the crack problem is reduced to the solution of integral equation (14). Various quantities u_0, u_1, L_i, M_i, N_i, P_i and v_0, v_1, Q_j, R_j, S_j, T_j can be expressed in terms of $\phi(x)$, as shown in the Appendix. One notices that the term related to κ is absent in Eq. (14). This emphasizes the indifference for SIFs between the plane-stress and the plane-strain elasticity under traction loading, although the displacement fields are different.

3.3 Stress-intensity factors

The Muskhelishvilli technique [10, Chapt. 6] can be used to explore the singular behavior of the solution of the integral equation (14) around the irregular points. For the edge cracks, the unknown function $\phi(s)$ may be expressed as

$$\phi(s) = \frac{\varphi(s)}{\sqrt{c-s}}, \tag{15}$$

where $\varphi(s)$ is a continuous and bounded function in the interval $0 \le s \le c$. We choose $\varphi(s)$ in a Taylor-series form, $\varphi(s) = \sum_{i=0}^{N} \gamma_i s^i$, in this paper. The SIF at the crack tip is

$$K_I(c_1) = -\sqrt{2a}\varphi(c). \tag{16}$$

To solve the integral equation (14), the following normalized variable is introduced in the interval $[0,\ c]$:

$$x = c(1+r)/2$$

and the following integration points are chosen as in [11]:

$$r_i = \cos\left(\frac{2i\pi}{2N+3}\right), \quad i = 1, 2, \ldots, N+1.$$

Equation (14) can be integrated by the Filon numerical integral method and a linear equation can be obtained. The problem is transformed to a series of linear equations about γ_i.

3.4 Validations

The results from the PM (present method) are validated in this section by the results from the FEM (finite-element method). A mesh configuration for the latter is shown in Fig. 3. Under different loads $p(x) = x^n$ ($n = 0, 1, 2, \ldots, 10$) and under different configurations (b and c), the normalized SIFs $k_I = K_I/\sqrt{\pi c_1}$ are obtained. Hereafter $N = 15$ is adopted in all demonstrative calculations. Table 1 indicates that good agreement has been achieved between the PM and the FEM, with the difference only occurring in the fourth significant digit.

3.5 Thermal stresses

The advantage of the present method lies in its versatility on handling arbitrary crack-face tractions. This capability is of particular relevance for the case of combined thermal/mechanical loading. To illustrate this point, attention is focused on the thermal-stress problem of a stack of periodically distributed edge cracks normal to the boundary of an infinite strip. In this section, we assume that both h_1 and h_2 tend to infinity. Accordingly, the temperature field is reduced to

$$\theta(x,t) = (1-x) - \sum_{i=1}^{\infty} \frac{2\sin(i\pi x)}{i\pi} \mathrm{e}^{-i^2\pi^2 t}. \tag{17}$$

The normalized temperature and stress distributions at different instants are shown in Fig. 4. Apparently, the thermal stress at a fixed location increases as the time increases; while the thermal stress at a certain time decreases as the

Table 1 Normalized SIFs of an infinite stack of periodically distributed edge cracks for different values of b and c under linear load $p(x) = x^n$

	N		0	1	2	3	4	5	6	7	8	9	10
$b = 0.5$	$c = 0.1$	FEM	1.039	6.492e − 2	5.061e − 3	4.279e − 4	3.773e − 5	3.411e − 6	3.137e − 7	2.919e − 8	2.741e − 9	2.592e − 10	2.466e − 11
		PM	1.039	6.493e − 1	5.061e − 3	4.280e − 4	3.773e − 5	3.411e − 6	3.137e − 7	2.919e − 8	2.742e − 9	2.593e − 10	2.466e − 11
	$c = 0.2$	FEM	0.8719	1.165e − 1	1.872e − 2	3.220e − 3	5.740e − 4	1.046e − 4	1.934e − 5	3.616e − 6	6.815e − 7	1.293e − 7	2.464e − 8
		PM	0.8719	1.165e − 1	1.872e − 2	3.220e − 3	5.740e − 4	1.046e − 4	1.935e − 5	3.616e − 6	6.816e − 7	1.293e − 7	2.464e − 8
	$c = 0.5$	FEM	0.5568	2.278e − 1	9.873e − 2	4.420e − 2	2.019e − 2	9.355e − 3	4.378e − 3	2.065e − 3	9.801e − 4	4.674e − 4	2.238e − 4
		PM	0.5569	2.278e − 1	9.873e − 2	4.420e − 2	2.020e − 2	9.356e − 3	4.379e − 3	2.065e − 3	9.802e − 4	4.675e − 4	2.239e − 4
$b = 5.0$	$c = 0.1$	FEM	1.112	6.791e − 2	5.233e − 3	4.395e − 4	3.857e − 5	3.476e − 6	3.189e − 7	2.962e − 8	2.778e − 9	2.624e − 10	2.493e − 11
		PM	1.112	6.792e − 2	5.234e − 3	4.395e − 4	3.858e − 5	3.477e − 6	3.189e − 7	2.963e − 8	2.778e − 9	2.624e − 10	2.493e − 11
	$c = 0.2$	FEM	1.095	1.344e − 1	2.078e − 2	3.495e − 3	6.142e − 4	1.108e − 4	2.034e − 5	3.780e − 6	7.091e − 7	1.340e − 7	2.547e − 8
		PM	1.095	1.345e − 1	2.078e − 2	3.496e − 3	6.143e − 4	1.108e − 4	2.034e − 5	3.780e − 6	7.092e − 7	1.340e − 7	2.547e − 8
	$c = 0.5$	FEM	1.063	3.311e − 1	1.286e − 1	5.426e − 2	2.388e − 2	1.078e − 2	4.950e − 3	2.301e − 3	1.080e − 3	5.102e − 4	2.425e − 4
		PM	1.063	3.311e − 1	1.287e − 1	5.427e − 2	2.388e − 2	1.078e − 2	4.950e − 3	2.301e − 3	1.080e − 3	5.103e − 4	2.425e − 4

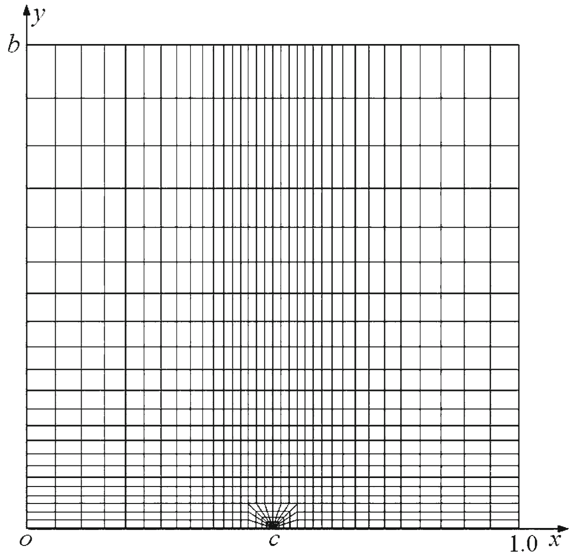

Fig. 3 The finite-element mesh

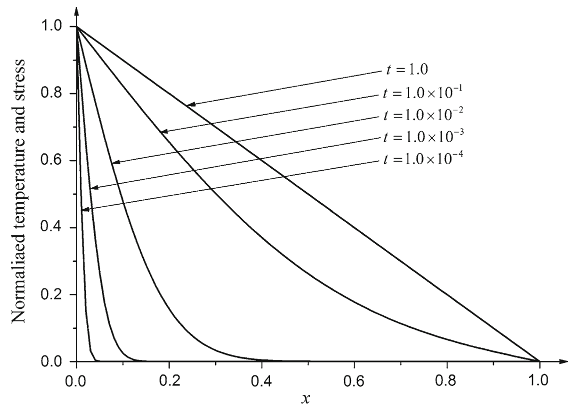

Fig. 4 Distributions of normalized temperature and stress at different instants t

distance from the crack-edge increases. After a period of time, the temperature distribution approaches a steady state, linear distribution with the x-coordinate.

For the convenience of the description, the normalized SIF is defined as follows:

$$k_I = K_I/\sigma_0^T\sqrt{\pi c_1}.$$

The normalized SIFs under only thermal loads are given in Fig. 5 for various crack lengths and separations. The following features are observed from Fig. 5.

1. After the thermal shock, the SIFs are initially very small since the temperature shock does not smear out in the length of strip. The normalized SIFs are plotted against the normalized time on a logarithmic scale ($\log_{10}(t)$) for different values of b and c. The normalized SIFs always approach the limits, set by SIFs under linearly loaded traction distribution, as the time parameter t exceeds that required to reach a steady state.
2. The normalized SIFs exhibit the same trend for different values of b, reflecting the vertical spacing between the neighboring cracks. All results reveal that the SIFs decrease as b decreases. That is the well-known shielding effect for parallel cracks. The present investigation confirms the effectiveness of shielding for the case of thermal-shock loading.
3. Also apparent in those figures is the influence of c on the SIFs. As c increases, the normalized SIFs decrease when b is less than a certain value (Fig. 5a–c). However, the abnormal phenomena will appear when b reaches that critical value (Fig. 5 (d)).

3.6 Application to linear load

The temperature-distribution equation (5) reveals that the temperature fields and the stress fields become steady and linear after a certain period of time. That provides a strong impetus to study the problem of linearly distributed load on the crack surfaces. Figures 6 and 7 illustrate the variations of SIFs for the crack-face loadings of $p_1(x) = 1.0$ and $p_2(x) = 1.0 - x$, respectively. The following features are observed.

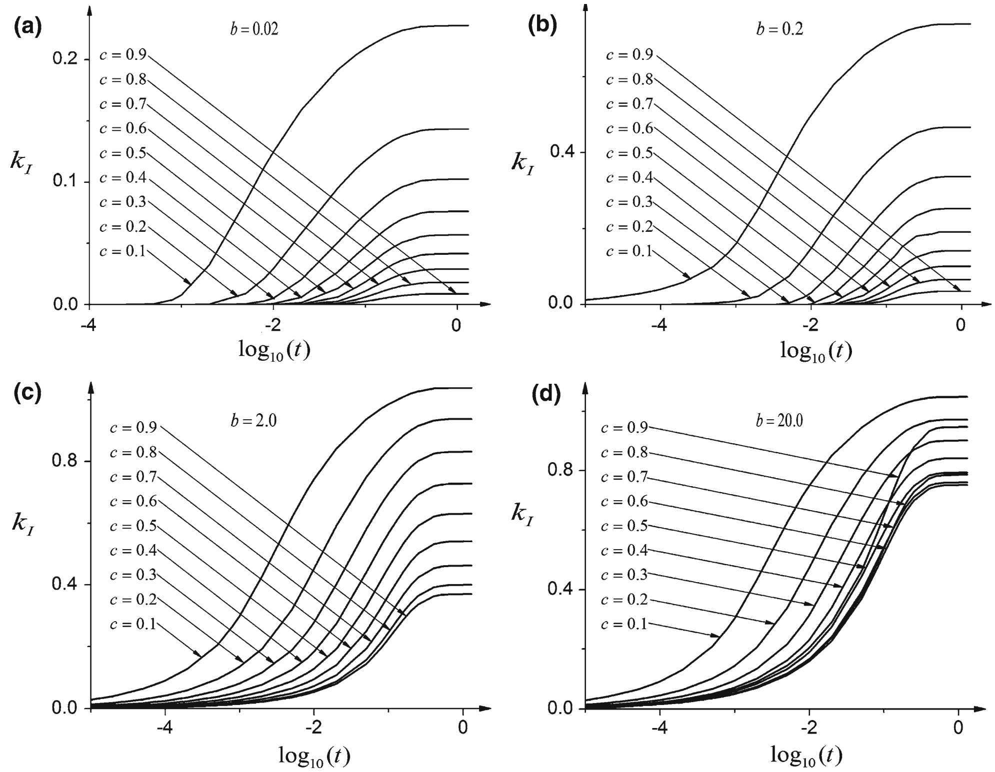

Fig. 5 Normalized SIFs of an infinite stack of periodically distributed edge cracks for different values of b, c and t

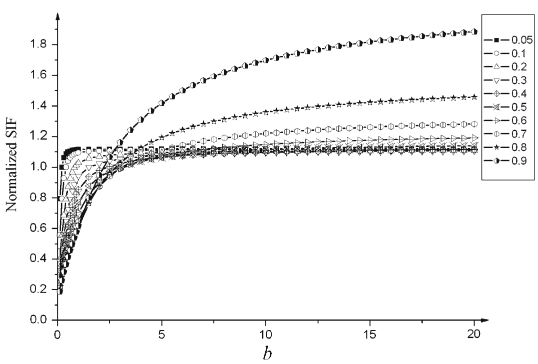

Fig. 6 Normalized SIFs of an infinite stack of periodically distributed edge cracks for different values of b and c under the uniform load $p(x) = 1.0$

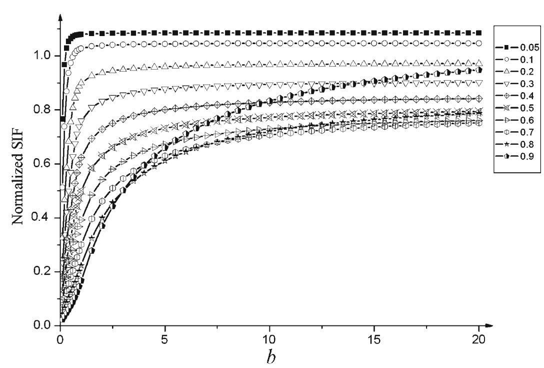

Fig. 7 Normalized SIFs of an infinite stack of periodically distributed edge cracks for different values of b and c under the linear load $p(x) = 1.0 - x$

Under a uniform load p_1, the normalized SIFs increase as b increases, which confirms the effectiveness of shielding. At the same b value, the SIFs decrease as c increases when $c \leq 0.5$, and increase as c increases when $c \geq 0.6$.

Under the linear load p_2, almost the same phenomena occur, except that the normalized SIFs at $c = 0.05$ are always larger than those at the other c-values plotted. As indicated by Fig. 4, the load p_2 corresponds to the limit case for the problem discussed in Sect. 3.5.

One may obtain the solution for an arbitrary linear loading by combining the above two results.

4 Conclusions

An infinite strip containing an infinite row of periodically distributed edge cracks normal to the edge has been studied. The configuration constitutes a simplified case of parallel shallow surface cracks exposed to thermal-shock loadings. An integral-transform technique was applied to characterize the transient SIFs under thermal-stress loading. A singular integral equation was deduced for general distributed pressure on the crack surfaces, and the results can be obtained through numerical solution to the singular integral equation.

An important feature of our work is to study the key dimensionless groups of different physical parameters. We normalized the problem so that the number of parameters are reduced. In the tested problem, the agreement between the results from the singular-integral-equation method and the FEM for different geometrical and loading parameters validates the accuracy of the integral-equation method. The transient normalized SIFs for edge cracks are given in Fig. 5, for various crack lengths and separations. The linear loads on the crack surfaces are discussed and the results are illustrated in Figs. 6 and 7. Although this thermo-mechanical crack problem has a simple geometry, the figures can be used, as a conservative estimate, to evaluate the severity of thermal shock in brittle materials by comparing the calculated SIFs with the fracture toughness of the material under consideration.

Acknowledgements The authors acknowledge the financial support from FRAMATOME-ANP.

Appendix: Expressions for various quantities in calculating the displacements

$$R_i = \frac{1}{\mu}\coth(bi\pi)\int_c^d \phi(t)\sin(i\pi t)\mathrm{d}t, \quad S_i = \frac{1-\kappa+4bi\pi\,\mathrm{csc}\,h(2bi\pi)}{2i\pi}R_i, \quad Q_i = \frac{(1-\kappa)}{2i\pi}\tanh(bi\pi)R_i,$$

$$T_i = -\tanh(bi\pi)R_i, \;\; P_j = \frac{4a^3b^2j^2}{\pi(b+2aj\pi\,\mathrm{csc}\,h(\frac{2j\pi}{b}))}\sum_{i=1}^{\infty}\frac{i\tanh(bi\pi)}{(b^2i^2+j^2)^2}R_i,$$

$$L_j = -\left(\frac{b(1+\kappa)}{2j\pi}+2\,\mathrm{csc}\,h\left(\frac{2j\pi}{b}\right)\right)P_j, \quad M_j = -\tanh(j\pi/b)P_j, \quad W_j = \frac{b(1+\kappa)}{2j\pi}\tanh\left(\frac{j\pi}{b}\right)P_j,$$

$$v_1 = \frac{(1+\kappa)}{4b\mu}\int_c^d t\phi(t)\mathrm{d}t, \quad u_0 = \frac{3-\kappa}{1+\kappa}v_1, \quad u_1 = -\frac{3-\kappa}{1+\kappa}v_1, \quad v_0 = -bv_1.$$

References

1. Nied HF (1983) Thermal shock fracture in an edge-cracked plate. J Therm Stresses 6:217–229
2. EI-Fattah A, Rizk A, Radwan SF (1992) Transient thermal stress problem for a cracked semi-infinite medium. J Therm Stresses 15:451–468
3. Nied HF (1987) Thermal shock in an edge-cracked plate subjected to uniform surface heating. Eng Fract Mech 26:239–246
4. EI-Fattah A, Rizk A (1993) A cracked plate under transient thermal stresses due to surface heating. Eng Fract Mech 45:687–696
5. Shindo Y, Atsumi A (1974) A laminate composite with an infinite row of parallel cracks normal to the interfaces. Int J Eng Sci 12:743–757
6. Shindo Y, Atsumi A (1975) Thermal stresses in a laminate composite with infinite row of parallel cracks normal to the interfaces. Int J Eng Sci 13:25–42
7. Davidson S (1993) The linear steady thermoelastic problem for a strip with collinear array of Griffith cracks parallel to its edges. J Eng Math 27:89–98
8. Qing H, Yang W (2005) Thermal shock analyses for a strip containing an infinite row of periodically distributed cracks normal to its edge. Theor Appl Fract Mec 44:249–260
9. Sternberg E, Chakravorty JG (1959) Thermal shock in an elastic body with spherical cavity. Q Appl Math 17:205–218
10. Muskhelishvili NI (1953) Singular integral equations. Noordhoff, Groningan
11. Erdogan F, Gupta GD, Cook TS (1973) Numerical solution of singular integral equations. In: Sih GC (ed) Mechanics of fracture. Noordhoff International Publishing, Leyden pp 368–425